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A MODEL FOR ANALYZING ENERGY IMPACT OF TECHNOLOGICAL CHANGE

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ABSTRACT

This paper describes the development of a linear model of the U. S. energy system. It is basically an input-output model that is tailored specifically for analysis of energy related problems.

The most significant feature is the development of a set of ficticious "energy product" sectors which define nonsubstitutable end-uses of energy. Thus instead of consuming fuels, the industrial sectors consume energy products (e.g. space heat, air conditioning, etc.). The advantage of this formulation is that it is no longer necessary to specify the production functions of each of the economy's many sectors to reflect fuel substitution possibilities. Technological changes associated with fuel substitution are localized in a small submatrix of the model instead of the entire rows corresponding to the energy sectors.

Next we examine the sensitivity of total energy demand to three types of technological change: changes in energy supply technologies, energy utilization efficiencies, and substitution among non-energy intermediate inputs. The coefficients describing these technologies fall in three distinct partitions of the matrix. Within each partition, the technological coefficients are ranked according to their importance relative to arbitrarily specified policy variables.

Finally, the accuracy of the model is analyzed by evaluating output tolerance for various values of the tolerance on its parameters.

1. INTRODUCTION

Most applications of the large scale input-output model of the U. S. energy-economic system [1], have been to assess energy impacts of changes in demands for final system outputs (e.g. as would result from car-bus substitution) [2-6]. Because of the necessary I-O assumption that the model's parameters* are constant, all early applications of that model were of an assessment, rather than predictive nature.

Recently, however, the model has been expanded and modified to accommodate projected future values of certain parameters important for energy policy analysis [7]. Some of these parameters, technical coefficients that would change as a result of fuel substitutions, are essentially the design parameters for the U. S. energy production system. Using the results of other models specifically designed to determine future values of these parameters [8], the I-O model may be partially updated to reduce the uncertainty of predictive results.

Updating the parameters specifying energy supply technologies is necessary in order to use a 1967 (the latest base year for which data are available) I-O model for predicting energy demand. However, it is not at all clear that updating only these parameters would be sufficient, for each of the model's 135,000 parameters affects the results directly or indirectly. Since each parameter specifies part of a production technology, each technological change (no matter how obscure) has an energy impact.

The purpose of this paper is twofold. First we review the rationale and development of the energy input-output model, with emphasis on the structural features that facilitate its applicability to problems of a predictive nature. Second we will discuss methods for quantifying the energy impact of technological change and demonstrate that these techniques can be employed to identify a subset of the model's parameters whose accurate updating would be sufficient to reduce the uncertainty of the model outputs.

Results of three calculations are presented in Section 4. The examples were selected to demonstrate the use of the model for quantifying impacts of technical change, identifying parameters to which model outputs are most sensitive, and tightening output error tolerances.

2. MODEL DEVELOPMENT

The model of ref. [1] defines and solves a system of N energy balance equations for each of the N sectors of the economy. Published data from the U. S. Department of Commerce allows implementation at a very detailed level, exceeding 360 sectors [9]. The model's derivation is described in refs. [1, 10] where detailed results are presented.

The model's parameters, elements of the matrix A of technological coefficients, are defined as follows:

$$A_{ij} = \frac{\text{amount of output from sector } i \text{ sold to sector } j}{\text{unit output from sector } j} \qquad (2\text{-}1)$$

Our values differ from the published values of ref. [9] for the base year 1967 in two ways. First, outputs of energy sectors are expressed in physical units (Btu) rather than current dollar values to account for the fact that energy is sold to different sectors at mostly different prices. Physical data are preferable for all sectors, but are not available. Second, sector outputs are domestic outputs only; this facilitates use of the results for analyzing energy impacts of foreign trade policies.

The energy costs of goods and services are given by the first five rows (corresponding to the five energy sectors) of the solution matrix $(I-A)^{-1}$. They are designated by the 5xN matrix ε_{kj} expressed in units of Btu's of energy type k required directly and indirectly to produce a unit of output from sector j for final consumption. The matrix $(I-A)^{-1}$ has the same units as A, shown in fig. 1 on the following page.

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* A matrix of coefficients fixing the production technologies of all sectors of the economy.

	Supplies	Industries
Supplies	$\frac{\text{Btu}}{\text{Btu}}$	$\frac{\text{Btu}}{\$}$
Industries	$\frac{\$}{\text{Btu}}$	$\frac{\$}{\$}$

Figure 1. Matrix of Technical Coefficients

2.1 Fuel Substitution Parameters

The most obvious drawback of this model was its inability to account for fuel substitution, and for the rapid changes in the technology of direct energy use brought about by recent substantial increases in fuel prices. Recognizing that fuels are highly substitutable for many purposes, we address that problem first by identifying a set of end uses: space heating, water heat, process heat, feedstocks, etc. To retain the advantages of the input-output formulation, we assume that while fuels may be substitutable, the end uses (called "energy products") are not. We accommodate this by adding a set of new sectors to this model, one corresponding to each energy product. The non-substitution assumption is reflected in the constant technological coefficients (Btu's of energy product/unit output from any sector).

	Supplies	Products	Industries
Supplies	$\frac{\text{Btu}}{\text{Btu}}$	$\frac{\text{Btu}}{\text{Btu}}$	0
Products	$\frac{\text{Btu}}{\text{Btu}}$	0	$\frac{\text{Btu}}{\$}$
Industries	$\frac{\$}{\text{Btu}}$	0	$\frac{\$}{\$}$

Figure 2. Expanded Matrix of Technical Coefficients

The A matrix corresponding to this new model is shown in Figure 2, where energy supply sectors (S) sell directly to the energy product sectors (P) which in turn distribute their outputs to the rest of the industries (I). Thus, by definition, there are no inputs of energy supplies to the industrial sectors; $A_{SI} = 0$, and $A_{PI} \neq 0$ instead. Since A represents only current account transactions, $A_{IP} = 0$ because energy cooperates with capital equipment rather than consumption goods, to produce energy products. Also note that $A_{PP} = 0$ by definition. It will be seen that none of these definitions are essential to the model; they may be relaxed later to accommodate special cases. They are described here only to highlight the relationships between the newly defined energy product sectors and the existing energy supply and industrial sectors.

To add these new sectors, it was necessary to derive the base year coefficients A_{PS} and A_{PI}. The method described in reference [11] was based on overall energy product control figures from reference [12] reconciled in each sector with actual fuel use data from reference [13]. As indicated in Table 1, energy supply sectors were expanded from 5 to 8 to distinguish technologies for electricity generation and to accommodate a coal gasification sector.

Supply Sectors	Product Sectors
Coal	Ore Reduction Feedstocks
Crude Oil & Gas	Other Feedstocks
High Btu Coal Gas	Motive Power
Refined Oil	Miscellaneous Thermal
Natural Gas	Water Heat
Fossil Elec.	Space Heat
Nuclear Elec.	Air Conditioning
Hydro Elec.	Miscellaneous Electric

Table 1. Energy Supply and Product Sectors

Notes

1. Electric supplies converted at 3413 Btu/kwh.
2. Motive power defined as energy at the drive shaft to allow for fuel and electric substitution within the model.
3. Miscellaneous thermal energy is that heat available for industrial processes or other uses.
4. Water heat is that transmitted to the water.
5. Space heat and air conditioning measure heat transferred to or from the building.
6. Miscellaneous electric measured at the wall socket. Includes all nonsubstitutable uses of electricity (motors, lighting, etc.).

In this new framework total direct and indirect requirements for energy supplies and products are given by the same equations derived in reference [1] for any given vector of final demands for goods and services Y.

$$X = \varepsilon Y \qquad (2\text{-}2)$$

where ε represents the supply and product rows of the matrix $(I-A)^{-1}$. These energy intensities are functions of A alone; the chief result of our redefinition of A is that parameters specifying fuel substitution technologies are now isolated in the small partitions A_{SS} and A_{SP}. These coefficients account for fuel substitution and may be computed exogenously as a function of relative prices, import quotas, and other factors, using models such as that of ref. [8].

One might expect, in addition, that these fuel substitution parameters would be among the most important with respect to the energy cost of goods and services. This expectation follows from the fact that A is well-conditioned and its eigenvalues are all less than unity, so the energy intensities can be expressed as a series expansion

$$(I-A)^{-1} = I + A + A^2 + A^3 + \ldots. \qquad (2\text{-}3)$$

In this equation the actual Btu content of the fuel appears in the first term, and the second term is the direct energy used in the last stage of the production process for each good or service. Non-energy technologies (such as substituting fiberglas for steel in cars) don't appear until the higher order terms. From this convergent series one might expect direct energy use technologies to be most critical, and that change in most non-energy technologies would be less important. This hypothesis will be tested in sec. 4.

2.2 Direct Energy Use Parameters

The parameters in the technological coefficient matrix that one would expect to be the next most important are A_{PS} and A_{PI}, the energy product rows. These reflect the efficiency of direct energy use in economic production processes.

Direct energy conservation measures implemented at the point of use, building insulation for example, can reduce the requirements for space conditioning represented by the coefficients A_{PI} in the production functions of each industry. Similar options exist for each of the other energy products.

These coefficients could change as a result of changes in the price of energy relative to capital or labor, or from a variety of conservation policies such as investment tax credits, subsidized loans for insulation programs, etc. Evaluation of the impact of such changes on total energy demand or on energy intensities of particular products is straightforward, using an efficient, inexpensive updating technique based on a square root-free Givens method for solving the system of equations

$$X = (I-A)^{-1}\, Y \qquad (2\text{-}4)$$

as described in reference [14].

2.3 Changes in Non-Energy Technologies

Certain other changes in non-energy technologies, elements of A_{IS} and A_{II}, may also substantially affect energy demands or intensities. As an example, observe that substitution of fiberglas for steel in auto manufacturing actually causes a shift in energy demand from coal to oil. This is because steel is coal-intensive and fiberglas is oil-intensive.

Technological changes of this type are usually accompanied by others in the same industry, or column of A. For example, the reduction of steel use would call for less welding and more epoxy bonding, which would alter other elements of the auto sector column.

Whether these technological changes, which affect only the higher order terms in eq. (2-3), are negligible depends on how they enter into the problem-specific importance functions defined in the next section.

3. ERROR BOUNDS AND PARAMETRIC STABILITY

As mentioned earlier, conventional I-O analyses are based on the assumption that all parameters, the matrix A of technical coefficients, are relatively stable over time. Since I-O data are typically six or seven years old when first published, and are only infrequently (once every five years) updated, some account must be taken of parametric uncertainty resulting both from measurement errors and from technical change over time. The longer the planning horizon for the predictive applications of the model, the greater the uncertainty. In this section we first discuss methods for estimating maximum error bounds on the results given uncertainty in A. Then we describe a technique for identifying a subset of A where technical changes or parametric uncertainty could have the greatest energy impact.

3.1 Maximum Error Bounds

Since stochastic error analyses are unwieldy for problems of this type and scale, we restrict our attention to estimates of maximum error bounds. A simple approach utilizing matrix norm analysis bounds the uncertainty on the inverse of a matrix due to an uncertainty in its elements as follows

$$\frac{\| C^{-1} - (C+\delta C)^{-1} \|}{\| C^{-1} \|} = \frac{\dfrac{\|\delta C\|}{\|C\|} \cdot M}{1 - M \cdot \dfrac{\|\delta C\|}{\|C\|}} \qquad (3\text{-}1)$$

provided

$$\|C^{-1}\| \cdot \|\delta C\| < 1$$

where M = the condition number of $C = \|C\| \cdot \|C^{-1}\|$. Eq. (3-1) says in effect that given a percentage (in the norm sense) perturbation on C, the resulting percentage perturbation on C^{-1} will be less than or equal to M times as large provided that $\|C\| \cdot \|\delta C\|$ is small. As demonstrated in section 4, this technique gives a very loose upper bound. An example will suffice here to show how the triangle inequality on which (3-1) is based could produce overly conservative results.

$$\text{Let } A = \begin{bmatrix} 5 & 0 \\ 0 & 200 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & .1 \end{bmatrix}$$

$$\|AB\| \le \|A\| \cdot \|B\|$$

$$20 \le 200 \cdot 1$$

This is clearly a very conservative bound.

A much tighter bound on the inverse uncertainty may be obtained using the following procedure, which involves creation of two perturbed A matrices, one which is the perturbation causing the greatest possible increase (i.e. positive tolerance) on all elements of $(I-A)^{-1}$ and the other which causes the greatest negative tolerance on $(I-A)^{-1}$. The inverses of these two matrices give the worst case plus and minus element-by-element tolerances on $(I-A)^{-1}$. It can be

proved [15] that for a wide spectrum of I-O models, these perturbed A matrices are easily created by permitting all elements of A to assume their maximum (minimum) values simultaneously. This gives a much tighter worst case bound than the norm bound, but it must be remembered that tightness of the bound will depend on the likelihood that all elements might be in error in the same direction simultaneously.

3.2 Identification of Important Parameters

To tighten the error bounds on results of any application of the model, one could update some or all of the model's parameters to reduce uncertainty. Using a method developed in ref. [15], we will define importance with respect to an 'importance function' of the general form:

$$J = f\,[\Delta(I-A)^{-1}, Y]$$

where J may be a scalar, vector, or matrix expression of the problem solution, and is of order less than or equal to that of $(I-A)^{-1}$. The notation $\Delta(I-A)^{-1}$ represents a perturbed inverse matrix, resulting from perturbations of the parameters A.

After specifying an uncertainty level on elements of A, one can evaluate the resulting ΔJ using the Sherman and Morrison relation [16]. A parameter A_{ij} is said to be *important* if its uncertainty causes some element of ΔJ to exceed a prescribed threshold. Its importance with respect to the entire J is quantified by

$$\sum_{mn}\sum \chi_{mn} \text{ where } \chi_{mn} = \begin{cases} \nu = \dfrac{\Delta J_{mn}}{\tau} \text{ where } \tau \text{ is the applicable importance threshold and } \nu \geq 1. \\ 0 \quad \text{otherwise.} \end{cases}$$

Note that a brute force application of the Sherman-Morrison relation will not suffice for large I-O models. If m, n, i, j $\epsilon\{1,2...,370\}$, 1.9 x 10^{10} tests would be required. Ref. [15] presents an efficient method for reducing the number of these tests needed to identify all i, j, m, n for which $\nu \geq 1$.

4. EXAMPLES

The model presented here is intended for predicting future demands for energy supplies and products as a function of magnitude of the GNP, the market basket of goods and services comprising it, and technology. While the overall energy demand may be predicted using simpler models, demand for specific energy supplies may not. Shifts among requirements for various types of energy supplies are highly sensitive to changes in technologies related to fuel substitution: changes that might be induced by supply constraints on certain resources, environmental regulations, taxes, or subsidies.

We shall first present a calculation showing the magnitude of such changes over a relatively short period of time. Then we focus on demand for a particular form of energy - electricity - and identify the parameters in the model to which electric demand is most sensitive. Finally, we use the model to predict demands for energy products and discuss the implications of accuracy in parameter estimation and the effects of technological change on the levels of these demands. All results presented here were obtained using a version of the model aggregated to 101 sectors, because more detailed data on energy products are only preliminary. However, all algorithms have been verified on similar models at the full 360-sector detail.

4.1 Requirements for Energy Supplies

Estimates of the technology for converting energy supplies to energy products in 1969 and 1985 (projected) were available from ref. [17]. These were derived from actual and expected values of supply, demand, and technological constraints acting on the U.S. energy supply system. These estimates were used to update the A_{SS} and A_{SP} partitions of the matrix of technological coefficients of our model. The total energy supplies required directly and indirectly to produce the actual 1967 GNP were then computed. This standard 1967 bill of goods (the latest available) was chosen so the results in Table 2 would reflect only evolution in the technology of producing energy products from energy supplies.

ENERGY SUPPLY	1969	1985	1969-1985 INCREASE
Coal	12,250	12,620	3%
Crude Oil & Gas	49,080	45,460	-7%
Hi Btu Coal Gas	0	490	--
Refined Oil	24,390	20,760	-15%
Natural Gas	19,630	20,070	-2%
Fossil Electric	3,614	2,781	-23%
Nuclear Electric	39	1,395	3,477%
Renewable Elect.	772	588	-24%

Table 2.

Energy Demands: 1969 vs. 1985 Energy Technologies

(Units: 10^{12} Btu)

It can be seen from the table that if the 1985 optimal energy supply technology were in place in 1969, consumption of various energy resources would have been substantially different. To accomplish a shift of this magnitude in the intervening sixteen years will evidently require that the nuclear power industry expand much more rapidly than the rest of the sectors, dominating an overall 10% increase in size of the electricity share. If capital shortages, safety or environmental problems, or other factors should act to retard expansion of the electric industry, conservation policies may be needed to close the gap between supply and demand. The effects of such electricity-conserving technical changes on electric demand are evaluated next.

4.2 Technical Change to Conserve Electricity

Here we select electricity demand as our importance function, J_E:

$$J_E = f\,[\,\Delta(I-A)^{-1}, Y]$$

$$J_E = u\cdot(I-A)^{-1}Y = u\cdot X$$

$$u_i = \begin{cases} 1 & i = \text{electric utilities} \\ 0 & \text{elsewhere} \end{cases} \qquad (4\text{-}1)$$

where u is a row vector which extracts and sums the total requirements for outputs from the electric utility sectors. Using the methods of sec. 3.2, the parameters to which this scalar function is most sensitive were determined. Table 3 ranks the most important technological coefficients in each of the major partitions of the matrix. Some of the non-energy technical coefficients are more important by this criterion than many direct energy use technologies, contrary to intuitive expectations (e.g. livestock → food is more important than air conditioning → wholesale and retail trade).

Table 3 identifies those technical coefficients where relatively small percentage changes result in the largest payoffs in electricity conservation. The technical feasibility of such changes and the likelihood of implementing them through various policies must be evaluated independently. The results here simply identify those areas where the feasibility of such policies should be examined.

Energy Supply Parameters

fossil electric	→ misc. electric
hydro electric	→ misc. electric
fossil electric	→ air conditioning
fossil electric	→ fossil electric including transmission losses)
fossil electric	→ water heat
fossil electric	→ cooking and refrigeration
fossil electric	→ space heat
refined oil	→ motive power

Energy Use Parameters

misc. electric	→ chemicals
misc. electric	→ primary nonferrous metals
misc. electric	→ primary iron and steel
misc. electric	→ wholesale and retail trade
air conditioning	→ wholesale and retail trade
misc. electric	→ medical, educational services
misc. electric	→ food
misc. electric	→ paper

Non-Energy Parameters

livestock	→ food
chemicals	→ plastics
stone and clay products	→ new construction
fabrics	→ apparel
heating equip.	→ new construction
chemicals	→ grain agriculture
nonferrous metals	→ new construction
printing	→ business services

Table 3. Parameters (Of Various Types) To Which Total Electric Demand is Most Sensitive

The model may also be used to estimate future demands for electricity, given the fact that the model's parameters are subject to some uncertainty over the planning horizon. It would be prohibitively expensive to accurately update and project future values of the more than 10,000 parameters of our 101-sector model; efforts must be concentrated on a few of the most important parameters. To help identify the point of diminishing returns (e.g. how many updated parameters is enough?), expected errors in the absence of parametric updating must be compared with those afterward. Below we describe a method for doing this based on analysis of maximum error tolerances.*

Let us assume the nominal values of the technical coefficients over the entire prediction period are given by the base year values, and let the uncertainty on all parameters be ± 10% over the time period of interest. We compute the Leontief inverses of the perturbed matrices and postmultiply by the actual base year final demand vector. We find that the perturbed matrices indicate only that the exact electric demand lies within an interval of +30.4% and -23.4% about the nominal value. While this tells us much more than the condition number criterion,** such error bounds are totally unsatisfactory for policy purposes.

If, however, we focused our attention on updating the most important 2% of the model's parameters, and were able to predict them 'exactly', our uncertainty would be reduced. To quantify the extent of that reduction, we set these parameters to their nominal values while perturbing the other 98% to their maximum upper (lower) bounds, and computed a new solution. The resulting 'predictions' of electric demand tightened the interval to only +4.7% and -4.2% about the nominal. This seems to be a quite satisfactory range of uncertainty in view of the rather conservative assumptions implied about the distribution (maximum upper bound) and additive nature of the assumed perturbations.

Those important parameters lying within the A_{SS} and A_{SP} partitions of the A matrix could be estimated with considerable certainty using specialized models such as the Brookhaven model, and recognizing that the long lead times associated with construction of energy supply facilities make predictions over a 10-year planning horizon relatively straightforward. Certain conservation options may be implemented between now and 1985 and could significantly affect total energy demand at that time. These should be explicitly recognized as these key parameters are updated.

The example described above was quantitatively unrealistic to the extent that the updated parameters cannot be known with absolute certainty. It shows, however, that confidence in the results can be substantially increased by updating only a very small fraction of the model's parameters. A budget to obtain a 'best estimate' could be most effectively spent in the manner dictated by these results.

4.3 Demand for Energy Products

Let J_P be a vector importance function representing the eight elements of the total requirements vector $X = (I-A)^{-1} Y$ representing demand for each of the energy products. The I-O model may be used to estimate future values of this importance function,

* Standard Monte Carlo-type statistical analyses of these problems are infeasible due to the size of the matrices and the nonlinearity of the matrix inversion step.

** The condition number of (I - A) is 49. This merely assured us that our 10% parametric errors might be magnified up to 49 times!

which in turn are input to the energy system optimization model of ref. [8] to determine optimal values of the parameters defining energy production technologies. Solution of the combined I-O and LP models proceeds in an iterative fashion, with energy supply system parameters and demand constraints (respectively) being updated at each step [7]. Here we shall examine the sensitivity of the importance function J_p to changes in key technological coefficients.

As in the previous example, the base year final demands are used to estimate the eight elements of the X vector corresponding to energy product demands. With all technological coefficients perturbed to their maximum upper or lower bounds ($\pm$ 10% as in all examples discussed here), the intervals for the results averaged about $\pm$ 18%. After identifying the most important 2% of the technical coefficients and holding them to their nominal values, the average was reduced to about $\pm$ 4%. Results for each of the eight energy service demands are shown in Table 4. Upper limits of interval for the +10% perturbation are shown; lower limits are smaller in all cases).

	Uncorrected	Corrected
Coke.............	32	6
Other Feedstocks..	27	4
Motive Power......	8	2
Process Heat......	30	7
Water Heat........	5	1
Space Heat........	9	3
Air Conditioning..	12	3
Misc. Electric....	20	6

Table 4. Energy Service Requirements: Maximum Upper Bounds (Percent) Before and After 'Correction' of Most Important Parameters

The most important parameters were determined using the methods of section 2, and the importance function J_p implicitly weighted all eight elements equally. The eight elements could have been differentially weighted if the analyst wanted to identify a different set of parameters that would further tighten the intervals on certain elements. For this specific application, specifying constraints for a linear programming model, a weighting scheme based on the LP shadow prices would perhaps be appropriate. The shadow prices finally resulting from the complete iterative solution of the I-O and LP models with updated coefficients would certainly be different from the roughly estimated weights initially employed, but these could be altered *a posteriori* if the resulting error bounds on the combined solution were unacceptable.

5. SUMMARY

An energy input-output model has been modified to facilitate its utilization for predictive applications. This was accomplished by defining a set of ficticious 'energy product' sectors corresponding to nonsubstitutable end uses of energy. One advantage of this new formulation is that it is no longer necessary to specify the production functions of the economy's many sectors to reflect fuel substitution possibilities. Parameters relating to fuel production and substitution technologies are now localized in a small submatrix in a form compatible with the outputs of other models specifically designed to project their future values. Methods were presented for identifying these and other parameters to which model outputs were most sensitive. Energy impacts of technological changes were quantified and example calculations demonstrated that prediction uncertainty could be reduced by as much as a factor of five through selective updating of a small (2%) subset of the model's parameters important to particular problems.

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